

Supplementary Information Appendix for the article:  
**Sideways walking: preferred is slow, slow is optimal,  
and optimal is expensive**

Matthew L. Handford and Manoj Srinivasan\*  
Mechanical and Aerospace Engineering,  
The Ohio State University, Columbus OH 43210, USA

The Supplementary Information has four components: (1) a Supplementary Information Appendix (namely, this document). (2) Experimental data, as a text file. (3) Video animation of the inverted pendulum walking, forward and sideways. (4) Computer programs which perform the model predictions, described below in greater detail.

**(1) Supplementary Information Appendix (this document).** This appendix has five sections S1 to S5 and four figures S1 to S4. The sections S1 to S5 respectively describe:

- **Section S1.** Comparison of other activities to sideways walking costs.
- **Section S2.** Mathematical models for walking metabolic costs.
- **Section S3.** Adding a leg swing cost and obtaining model-based optimal speeds.
- **Section S4.** Covariances and error estimates for the metabolic rate fits.
- **Section S5.** Metabolic cost of walking at zero speed versus cost of resting.

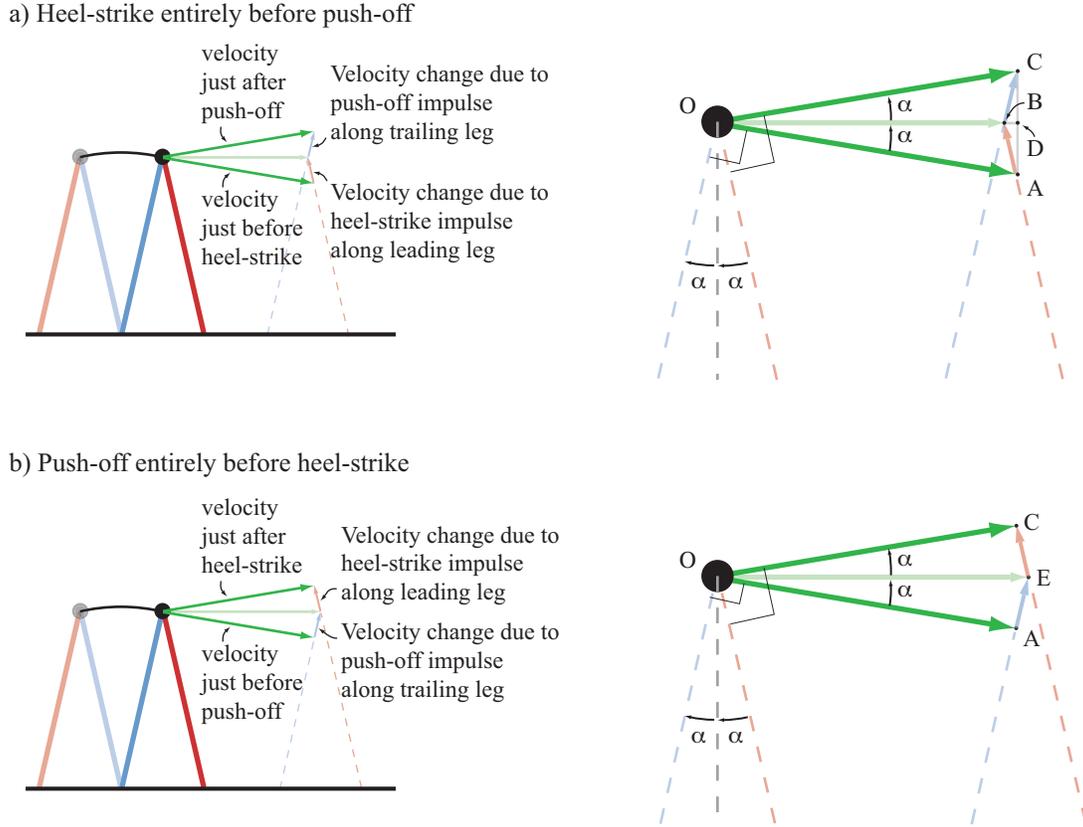
**(2) Experimental data.** Text file containing all (de-identified) human subject data used in this article.

**(3) Video animation.** Two video animations: one shows the simplest model of forward walking, namely inverted pendulum walking and the second shows our simple model of sideways walking. For sideways walking, the momentary rest phase when the legs are vertical is exaggerated in the sideways walking animation so as to make its presence more clear; in the model, this phase is of infinitesimal duration. The leg swing shown corresponds to a simple pendular motion, not included in the simulated equations of motion.

**(4) Computer programs.** A text file containing MATLAB programs. These are used to simulate the mathematical models for sideways walking and compute its cost. One sub-folder contains calculations of metabolic cost with fixed step length and no leg swing cost and another sub-folder contains calculations with leg swing cost, without step length constraints.

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\* Author for correspondence: Manoj Srinivasan, [srinivasan.88@osu.edu](mailto:srinivasan.88@osu.edu), <http://movement.osu.edu>



**Figure S1: Impulsive step-to-step transition.** a) Heel-strike entirely before push-off. b) Push-off entirely before heel-strike. The transition from an inverted pendulum motion about one leg to an inverted pendulum motion about the next leg is accomplished by two impulses in sequence: a heel-strike impulse from the leading length and a push-off impulse from the trailing leg. These impulses re-direct the center of mass velocity from downward ( $\vec{OA}$ ) to upward ( $\vec{OC}$ ). The panels on the right show a zoomed-in version of the step-to-step transitions. The velocity changes due to heel-strike are shown in light red ( $\vec{AB}$  and  $\vec{EC}$ ), velocity changes due to push-off are shown in light blue ( $\vec{BC}$  and  $\vec{AE}$ ), the horizontal velocity between the two impulses is shown in light green ( $\vec{OB}$  and  $\vec{OE}$ ), the direction of the two legs are shown as dashed lines.

## S1 Comparisons of other activities to sideways walking costs

**Forward walking and running.** At low speeds, the sideways walking cost is much higher than the forward walking cost: in particular, the gross sideways cost (including resting cost, per distance or per time) is about twice forward walking cost at 0.6 m/s. Also, we find that walking sideways at 1 m/s has the same gross metabolic rate as running forward at about 2.3 m/s (using [1]).

**Carrying a load.** Forward walking costs can also be increased by carrying a load. Up to carrying an extra load of about 75% body weight, gross metabolic rate has been shown to increase roughly proportional to total mass at a given speed [2]. Extrapolating this cost increase, one might have to carry about 100% body weight while walking forward to have the same gross metabolic cost as sideways walking at 0.6 m/s.

**Small animals.** Per unit mass and for a given absolute speed, small animals have a much higher metabolic rate than larger animals. The allometric scaling of net metabolic cost per distance per unit mass with the animals’ body mass  $M_{\text{animal}}$  is roughly like  $M_{\text{animal}}^{-0.3}$  [3]. Net sideways walking metabolic cost per distance is three to five times (say, four) higher than forward walking. So, an animal that is roughly  $4^{(1/0.3)} \approx 100$  times less massive than a human, that is, about a kilogram (in order of magnitude), might be expected to have the same cost per distance per unit mass. Note that the  $M_{\text{animal}}^{-0.3}$  cost-dependence power law was constructed with mostly running data [3], so the above estimate may change slightly when re-done with walking allometric data.

**Robots.** Finally, despite the high metabolic cost of sideways walking compared to normal human walking, many state-of-the-art legged walking robots are much worse than even sideways walking; for instance, the two legged Asimo robot is about ten times more expensive than normal human walking (e.g., as estimated by [4]).

## S2 Mathematical models for walking metabolic costs.

**Work-based metabolic cost.** In this article, for simplicity, we only use a work-based metabolic cost. That is, if  $W_p$  was the total positive work and  $W_n$  was the total negative work over a single step, we model the metabolic cost over the step by  $b_1 W_p + b_2 W_n$ . Here,  $b_1 = 4$  and  $b_2 = 1$  are the (approximate) reciprocals of muscle efficiencies for positive and negative work respectively [5, 6]. If  $T_{\text{step}}$  is the duration of a single step, the metabolic rate is computed as  $(b_1 W_p + b_2 W_n) / T_{\text{step}}$ .

**Stance cost.** The metabolic cost of the step-to-step transition and the cost of starting and stopping at mid-stance together form the **stance cost**  $\dot{E}_{\text{stance}}$ . This is the metabolic cost due the work performed by the stance leg (the leg in contact with the ground) [7]; this cost is sometimes dubbed as being due to “center of mass work” and is similar to the so-called “external work” [8]. In particular, this cost ignores the swinging the leg about the upper body.

For the sideways walking model in the main manuscript and for the forward inverted pendulum walking models, the step-to-step transition is accomplished using an impulsive push-off from the trailing leg and an impulsive heel-strike from the leading leg. As shown in Figure S1, these two impulses change of the direction of the hip velocity from downward at the end of one inverted pendular phase to upward at the beginning of the next pendular phase. The work done by the two impulses depend on the order in which the impulses occur and more generally, on the amount of overlap they have. Work expressions for arbitrary overlap are derived in [5], with simplifications for the limit of small leg angles during walking. In this section, for completeness, we derive the formulas for the positive and negative work performed by the impulses, without the small angle approximation, adapted from [9].

**Heel-strike entirely before push-off.** In Figure S1a,  $\overrightarrow{OA}$  is the velocity just before heel-strike at the end of the step.  $\overrightarrow{OC}$  is the velocity at the beginning of the next step, after both heel-strike and push-off are complete. A heel-strike impulse along the leading leg produces a velocity change  $\overrightarrow{AB}$  along the leading leg, resulting in a horizontal post-heel-strike velocity  $\overrightarrow{OB}$ . Then, the trailing leg pushes off, resulting in a velocity change  $\overrightarrow{BC}$  along the trailing leg, thereby changing the center of mass velocity from  $\overrightarrow{OB}$  to  $\overrightarrow{OC}$ .

By symmetry, the negative work done by the heel-strike is equal to the positive work done by the push-off. Because the sign of the work is fixed for each impulse (entirely positive or entirely

negative), the work can be calculated as the change in kinetic energy between  $\vec{OA}$  or  $\vec{OC}$  and  $\vec{OB}$ .

$$|\text{Work}| = \frac{1}{2}m|\vec{OA}|^2 - \frac{1}{2}m|\vec{OB}|^2 = \frac{1}{2}m|\vec{OC}|^2 - \frac{1}{2}m|\vec{OB}|^2. \quad (1)$$

Here,  $\vec{OB} = \vec{OD} - \vec{BD}$ , where  $|\vec{OD}| = |\vec{OA}| \cos \alpha$  and  $|\vec{BD}| = |\vec{AD}| \tan \alpha = |\vec{OA}| \sin \alpha \cdot \tan \alpha$ . Therefore,

$$|\vec{OB}| = |\vec{OA}| (\cos \alpha - \sin \alpha \cdot \tan \alpha) = |\vec{OA}| \frac{\cos 2\alpha}{\cos \alpha}.$$

Thus,

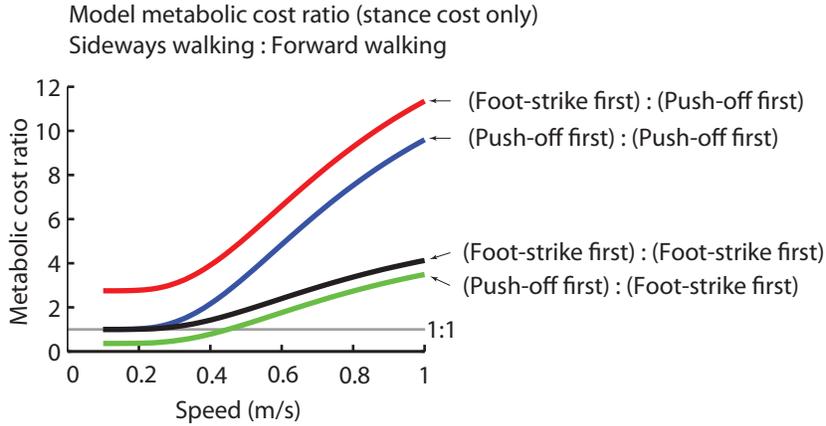
$$|\text{Work}| = \frac{1}{2}m|\vec{OA}|^2 \left( 1 - \frac{\cos^2 2\alpha}{\cos^2 \alpha} \right), \quad (2)$$

as noted in the main manuscript, where we use the notation  $v^- = |\vec{OA}|$ .

**Push-off entirely before heel-strike.** In Figure S1b,  $\vec{OA}$  and  $\vec{OC}$  are identical to those in Figure S1a. A push-off impulse along the trailing leg produces a velocity change  $\vec{AE}$  to bring the velocity to  $\vec{OE}$ . Next, a heel-strike impulse along the leading leg produces a velocity change  $\vec{EC}$ , bringing the velocity to  $\vec{OC}$ .

Again, because each impulse performs work of a fixed sign (entirely positive or entirely negative), the work done can be computed as the kinetic energy difference between  $\vec{OA}$  or  $\vec{OC}$  and  $\vec{OE}$ :

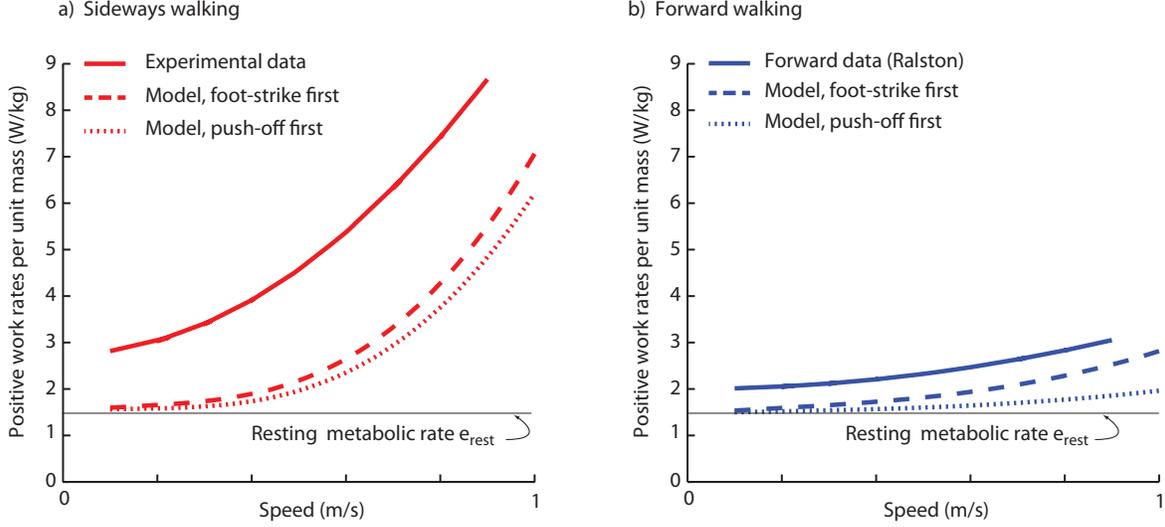
$$|\text{Work}| = \frac{1}{2}m|\vec{OE}|^2 - \frac{1}{2}m|\vec{OA}|^2 = \frac{1}{2}m \left| \frac{\vec{OA}}{\cos \alpha} \right|^2 - \frac{1}{2}m|\vec{OA}|^2 = \frac{1}{2}m|\vec{OA}|^2 \tan^2 \alpha. \quad (3)$$



**Figure S2: Metabolic cost ratios.** Ratios of metabolic cost between sideways walking and forward walking models each with either foot-strike (heel-strike) first or push-off first. Velocities range from 0 to 1 m/s and step length is equal to 0.5 m. The ratios are the same at a given speed whether one uses metabolic cost per time or metabolic cost per distance. These metabolic cost estimates are based on the stance work alone; they do not have a leg swing cost or added resting cost.

**Computing  $v^- = |\vec{OA}|$ .** Computer programs (in MATLAB) that compute  $v^-$  as a function of forward speed  $v$  and step length  $d_{\text{step}}$  are part of the Supplementary Information. As noted in the

Experimental Data versus Model predictions (with resting cost added, no leg swing cost)



**Figure S3: Comparison of experiment and stance cost model.** The experimental metabolic rate  $\dot{E} = a_0 + a_2 v^2$  is compared with the stance work-based metabolic rate predicted by the model with resting cost added:  $\dot{E}_{stance} + e_{rest}$ , for a) Sideways walking and b) Forward walking. The models had a constraint on step length that did not exist in the experiment and did not involve any cost for leg swing.

main manuscript, we compute the  $v^-$  as that required to achieve the correct average forward speed over the inverted pendular phase (see also [9]), as follows.

For a given  $\dot{\theta}(0^+)$ , starting at the vertical position  $\theta = 0$ , we integrate the pendulum differential equations  $\ddot{\theta} + (g/\ell)\theta = 0$ , until  $\theta$  becomes consistent with half-step length  $d_{step}/2$ . i.e., the integration is stopped when  $\theta = \sin^{-1}(d_{step}/2\ell)$ . The time at the end of integration is  $t = T_{step}/2$  and the corresponding body speed just before step-to-step transition is  $v^- = \ell\dot{\theta}(T_{step}/2)$ . We compute corresponding average speed is  $v_{avg} = d_{step}/T_{step}$ . We determine the  $\dot{\theta}(0^+)$  at initial vertical position so that the computed average forward speed is as desired:  $v_{avg} = v$ , accomplished by solving a root-find problem with MATLAB's `fsolve`. See appended supplementary programs.

Note that, in our models, the speed  $v^-$  just before the step to step transition is identical for all four gaits. The sideways walking gait has the same  $v^-$  as the forward walking gaits because the stopping and starting at mid-stance for sideways walking is assumed to take infinitesimal time, while the rest of the inverted pendular motion is identical in all gaits considered.

**Stopping and starting at mid-stance.** In addition to the work of the step-to-step transition, the sideways walking gait has an additional cost due to coming to rest and accelerating back to appropriate speed, when the legs are in the vertical position. The positive and negative work both equal  $m\ell^2\dot{\theta}(0^+)^2/2$ , the kinetic energy just after the hip impulse brings the body to the necessary speed.

**Stance cost comparisons between sideways and forward walking.** To compare the costs of sideways and forward walking, we simply compare their stance costs, for given speed  $v$  and fixed step length  $d_{step}$ . We consider four different gait variations: two sideways walking gaits and two forward walking gaits, one with foot-strike/heel-strike before push-off and another with push-off before heel-strike. For these four gait variations, Figure 2b of the main manuscript shows these

work-based metabolic rate estimates for a fixed step length  $d_{\text{step}} = \ell/2$  and  $\ell = 1$  m. The same ordering of the four costs is found at all other step lengths explored as well ( $0 < d_{\text{step}} < 1$ ). Figure S2 shows the ratio of sideways to forward walking stance metabolic rates, considering both impulse sequences for each gait.

We ignore the leg swing cost in this comparison for the following reason. In a single step of forward walking, only one of the two legs swings forward by the angle consistent with the step length. In a single step of sideways walking, both legs take turns swinging forward, but only for half the step length each. Thus, at a given speed and step length, the leg swing cost in sideways and forward walking are likely to be comparable, with sideways walking leg swing cost being larger (by a factor of two according to the simple kinetic energy based leg swing cost model described below). Thus, ignoring the leg swing cost is likely to underestimate the ratio of sideways and forward metabolic costs.

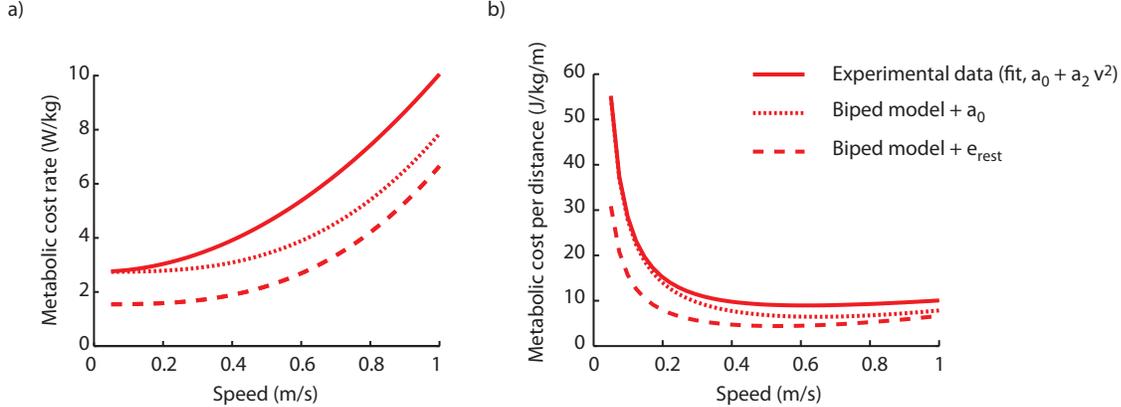
**Stance cost comparison with experimental metabolic data.** Figure S3 compares the experimental metabolic rate, with the model-based stance metabolic rate (again, for  $d_{\text{step}} = \ell/2$  and  $\ell = 1$  m). We add  $e_{\text{rest}}$  to the model, as the model does not have a resting metabolic rate. We see that the explanatory power of the stance cost of this simple model is about the same for both sideways walking and forward walking.

### S3 Adding a leg swing cost and obtaining model-based optimal speeds

Adding further terms to the stance work-based metabolic cost model allows us to not constrain the step length and also predict meaningful (non-zero) optimal speeds. The simplest modification involves the addition of two terms (1) a leg swing cost and (2) a constant offset cost ( $a_0$  or  $e_{\text{rest}}$ ). Note that for the calculations in this section, we used foot-strike entirely before push-off, as this choice gives the higher costs among the two impulse sequences.

**Leg swing cost.** First, to be able to predict step lengths, our biped model needs a leg swing cost, as described in [9, 10]. Even though [10, 11] have argued that a force-based leg swing cost fits data slightly better than a work-based leg swing cost, for simplicity and consistency, we use a work-based leg swing cost. In particular, we use the simplest possible additive leg swing cost as follows. On average, the foot has to move with the body at speed  $v$ , but it comes to rest twice every step. That is, positive work is done on the leg/foot to accelerate it, twice every step, so that the foot has the same average speed as the body. We approximate this work as  $0.5m_{\text{foot}}v^2$ , as in [9, 12], where  $m_{\text{foot}} = 0.0435m$ , so as to match the moment of inertia of the leg [13] with a mass at the foot. Now, instead of a root find procedure, we use numerical optimization to obtain the midstance  $\dot{\theta}(0^+)$  and the step time  $T_{\text{step}}$  so that the biped travels at a given speed  $v$  and has the least metabolic rate,  $\dot{E}_{\text{stance}} + \dot{E}_{\text{swing}}$ , defined as the cost over a step divided by  $T_{\text{step}}$ . This model-predicted metabolic rate goes to zero as  $v \rightarrow 0$ . Therefore, we add  $e_{\text{rest}}$  to this cost when comparing with experimental data in Figure S4 i.e., we plot  $e_{\text{rest}} + \dot{E}_{\text{stance}} + \dot{E}_{\text{swing}}$ . Because the experimental metabolic rate has a zero-speed cost  $a_0 > e_{\text{rest}}$ , we also show comparisons in Figure S4 with  $a_0 + \dot{E}_{\text{stance}} + \dot{E}_{\text{swing}}$ .

**Optimal speeds from model.** As can be gleaned from the metabolic cost per unit distance curves of Figure S4b, the optimal speeds obtained from the model with leg swing cost are 0.5319 m/s when  $e_{\text{rest}}$  is added and 0.6420 m/s when  $a_0$  is added.



**Figure S4: Biped model with leg swing cost.** a) Metabolic rate from experiment and biped model with an additional leg swing cost. We show two versions of model predictions, one with resting cost  $e_{\text{rest}}$  added and another with zero-speed cost  $a_0$  added. The latter is closer to experimental data. b) Metabolic cost per unit distance for experiment and biped model, again using two different offsets  $e_{\text{rest}}$  and  $a_0$ . These curves are obtained by dividing the corresponding curves in panel-a by speed  $v$ . This panel is identical to Figure 2c of the main manuscript. All model predictions assumed push-off before heel-strike. The step length was not constrained; at each speed, the step length was selected so as to minimize  $\dot{E}_{\text{stance}} + \dot{E}_{\text{swing}}$

**Metabolic cost is underestimated by the model.** As seen in Figure S4, the metabolic cost is underestimated by the model. Perhaps this underestimation is partly explained by force-related cost terms e.g., [11, 14, 15], partly due to the fact that the step length cannot go to zero for real human walking because of finite hip width. Also, the model is vastly simpler than the human body, with fewer degrees of freedom. Finally, the model assumes ideal impulsive push-off and foot-strike of infinitesimal duration; it is shown in [6, 16] that spreading these impulses over non-infinitesimal time durations, with smooth and finite ground reaction forces, will increase the cost substantially; after all, the ideal impulsive model was shown to be energy optimal in [17], as also elaborated at the end of this section.

Of course, even the fraction of the metabolic cost that we have explained using mechanical work could be partly due to force-related terms if muscle positive and negative work could be replaced by tendon positive and negative work.

Predicting the metabolic cost as a function of speed without actually fitting the model to data remains even an open problem for forward walking. Earlier detailed attempts to explain metabolic cost trends (e.g., [18]) have fallen short of quantitative predictions, or have required some fitting of model to data (e.g., [10] for the step length dependence). For instance, the articles that explain the metabolic cost as a function of speed using a force-based metabolic cost, while quite successful, have all only found correlations [11, 14, 19] or used fitting to data, rather than explaining the actual magnitudes through a priori models that start from a basic understanding of how muscles work. In contrast, our metabolic cost comparisons involve no fitting.

In future work, we hope to improve upon this work to predict and explain the observed metabolic cost variations and be consistent with as much extant data. With a force-plate measurement of ground reaction forces for each foot, we could estimate the work performed by the legs (using the so-called “external work” calculations [20, 21]) and compare with those predicted by our models. We could also use such force measurements to determine the sequence or overlap of push-off and foot-strike. Measurement of step length and step periods would also let us compare them with model predictions, as well as to use them to constrain the models better.

**Why did we choose impulsive walking strategies?** Our sideways walking model consists of passive inverted pendulum motions interrupted by discrete impulses, for instance, foot-strike, push-off, and hip torque impulses, which perform finite work in infinitesimal time. Similarly, the forward walking model consists of passive pendulum motions interrupted by heel-strike and push-off impulses. Why did we choose these specific walking models, as opposed to other simpler model choices such as, say, passive dynamic walking models [22]?

First, given a point-mass biped model with extensible legs, capable of an infinite variety of walking motions, it has been shown (in [6, 9, 17]) that the optimal forward walking gait minimizing a work-based metabolic cost is the inverted pendulum walking motion with push-off and heel-strike impulses. Similarly, swinging a leg (pendulum) or moving a mass from one position to another can be accomplished with least work by an impulsive strategy for the torque or force, as demonstrated in [9] and the appendix of [6]. Thus, in this article, we have used these impulsive sideways and forward inverted pendulum walking gaits, because such impulsive gaits are known to minimize work-based metabolic cost models for related more general biped models.

Note that the sideways walking described here cannot be achieved without hip torques, with just push-off and heel-strike. In our model, the hip-torques arrest the forward speed when the legs are together and vertical, and restart the motion again. When the legs are together, the push-off-heel-strike along the legs have no ability to kill the forward speed, because the legs are perpendicular to the forward body speed.

Another class of simple walking models is the so-called **passive dynamic walking** models (e.g., [22]), in which a completely passive biped (no muscles or motors) walks downhill under the power of gravity. The main difference between our models and these slope-walking passive dynamic walking models is that passive dynamic walking replaces the push-off or other positive work impulses by gravity and the leg swing is entirely passive. This replacement also means that passive dynamic walking is limited to walking on shallow slopes; it is physically infeasible to model uphill walking, for instance, by such passive walking. Similarly, for biped models with point-mass and point-feet (as here), we do not find any passive walking motions similar to sideways walking. Sideways walking involves coming to rest with both feet together every mid-stance; starting from such a rest state on the slope leads to the walker falling down or not moving, without additional forcing (like hip torques).

## S4 Covariances and error estimates for the metabolic rate fits

**Fits to pooled data.** For metabolic rate data pooled over the subject population, we obtained  $a_0 = 2.742$  W/kg and  $a_2 = 7.313$  W/(m s<sup>-1</sup>)<sup>2</sup>/kg. The error covariances between the coefficients were obtained via a bootstrap re-sampling procedure (see [23]).

This bootstrap re-sampling procedure involves resampling from the 80 data points of  $(v, \dot{E})$  across the 10 subjects, thereby creating new 80-data-point data sets (called bootstrap samples), and re-computing the regression coefficients for each such bootstrap sample. The error covariances estimated from this distribution of regression coefficients were  $\sigma_{00}^2 = 0.015$ ,  $\sigma_{22}^2 = -0.033$  and  $\sigma_{02}^2 = 0.13$  in appropriate units. The standard deviations of the respective coefficients may be obtained by taking the square root of the variances  $\sigma_{ii}^2$ , respectively  $\sigma_{00} = 0.123$  W/kg and  $\sigma_{22} = 0.36$  W/(m s<sup>-1</sup>)<sup>2</sup>/kg.

A distribution of optimal speeds  $v_{\text{opt}}$  was also determined from the bootstrap distribution of regression coefficients, giving a standard deviation of 0.027 m/s and a 95% confidence interval of 0.495-0.754 m/s, serving as error estimates for the computed  $v_{\text{opt}}$ .

**Fits to individual data.** By fitting the equation to each individual’s metabolic data and then averaging the obtained coefficients, we get roughly the same average coefficients:  $a_0 = 2.746$  W/kg and  $a_2 = 7.306$  W/(m s<sup>-1</sup>)<sup>2</sup>/kg, with covariances between the different subjects’ coefficients being  $\sigma_{00}^2 = 0.38$ ,  $\sigma_{22}^2 = 0.56$  and  $\sigma_{02}^2 = 0.252$  in appropriate units.

**Fits with a linear term.** We obtained roughly the same optimal speeds when we used a more general regression equation, now including a linear term (although not conventional in the literature [24, 25]):  $\dot{E} = b_0 + b_1 v + b_2 v^2$ . The best-fit coefficients were  $b_0 = 2.4622$  W/k,  $1.1791$  W/(m s<sup>-1</sup>)/kg, and  $b_2 = 6.2715$  W/(m s<sup>-1</sup>)<sup>2</sup>/kg, and the optimal speed  $v_{\text{opt}} = \sqrt{b_0/b_2} = 0.6266$  m/s, compared to  $\sqrt{a_0/a_2} = 0.6124$  m/s. See also section S5.

## S5 Metabolic cost of walking at zero speed versus cost of resting

We note that the metabolic rate of sideways walking at zero speed, namely  $a_0 = 2.742$  W/kg, is strictly greater than the resting (sitting) metabolic rate  $e_{\text{rest}} = 1.542$  W/kg; in fact, this is almost an 80% increase over sitting metabolic rate, as a percentage of metabolic rate. Is this substantial difference a real phenomenon, or an artifact of data processing or assumptions?

First, note that the metabolic cost of standing rest is only about 13% on average higher as opposed to sitting rest [26]. Thus, that walking very slowly requires standing cannot explain the cost increase.

Second, as noted in the main manuscript, a similar but smaller difference (about 30-40% over) is observed in regular forward walking, as already established in early work on walking [24] and repeated in subsequent studies [25]. Because  $a_0$  is obtained by extrapolating the equation obtained by fitting to metabolic measurements at speeds substantially different from zero, one might suspect that the  $a_0 - e_{\text{rest}}$  is an extrapolation artifact. However, Ralston already addressed this suspicion by explicitly performing experiments of subjects walking very close to zero speed (“as slowly as possible compatible to normal balance”) and observing that there was about a 33% difference over standing cost, and the cost he obtained for very slow walking was indeed close to his regressed  $a_0$ .

Thus, very slow walking seems qualitatively different from standing still. Perhaps the increased cost is due to stability issues or due to the cost of switching weight between the feet. The reasons for this difference have not been established in the literature. Independent of the reason, this difference seems a real phenomenon, as opposed to being a data processing artifact. This difference has real consequences to behavioral predictions based on metabolic energy optimality (see also [1, 27] for similar discussions).

Note that while we get a lower zero-speed cost  $b_0 = 2.46$  W/kg when using a different regression model, now with an additional linear term, this cost is still significantly above  $e_{\text{rest}}$ . In any case, as noted above, Ralston [24] showed that  $a_0$  approximates the zero-speed cost well, at least in forward walking.

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