Model Description

The model structure is outlined in Fig. S1. Throughout, $i$ indexes individual soldiers.

**Face Width-Height Ratio**

The measurements of face width-height ratio ($R(i,m)$) are subject to measurement error, so they are modelled as a function of the “true” face width-height ratio ($F(i)$):

$$R(i,m) \sim N(\omega(i), \sigma_{FWHR}^2)$$  \hfill (S1)

where $\sigma_{FWHR}^2$ is the observation variance. $\omega(i)$ is the expected measurement, which includes the effect of smiling, as well as the true FWHR:

$$\omega(i) = F(i) + \nu_{SMILE}(M(i) - \bar{M})$$  \hfill (S2)

$M(i)$ is an indicator for smiling (i.e. $M(i)=1$ if the soldier smiled, 0 if not; $\bar{M}$ is thus the proportion of soldiers who were smiling). We assume that the true face width-height ratios come from a common distribution:

$$F(i) \sim N(\mu_F, \sigma_F^2)$$  \hfill (S3)

**Survival of Winter War**

We assume that the event that soldier $i$ survived the winter war, $S(i)$, follows a Bernoulli distribution with probability $\pi(i)$, and then use a standard logistic regression model with covariates FWHR ($F(i)$), rank ($R(i)$), regiment ($G(i)$), and birth place ($B(i)$):

$$\log\left(\frac{\pi(i)}{1-\pi(i)}\right) = \beta_0 + \beta_{FWHR}(F(i) - \mu_F) + \beta_{RANK}(R(i)) + \beta_{REG}(G(i)) + \beta_{BP}(B(i))$$  \hfill (S4)

where $\beta_0$ is the intercept, $\beta_{FWHR}$ the regression coefficient for FWHR, and $\beta_{RANK}$, $\beta_{REG}$, and $\beta_{BP}$ are coefficients for the effects of the categorical variables rank, regiment and birth place respectively.

**Number of Children**

We assume that the total number of children fathered by soldier $i$, $N_T(i)$, follows a Poisson distribution with mean $\lambda(i)$:

$$N_T(i) \sim \text{Poisson}(\lambda(i))$$  \hfill (S5)

and that the number of these children that were born before the war, $N_B(i)$ follows a binomial distribution:

$$N_B(i) \sim \text{Binomial}(N_T(i), (1-S(i)) + S(i) \nu_{i})$$  \hfill (S6)

The probability $S(i) + (1-S(i))\nu(i)$ reflects the fact that if the soldier died, this has to be 1. $\nu(i)$ is then modelled using a logistic regression against mean-centred age:

$$\log\left(\frac{\nu(i)}{1-\nu(i)}\right) = \alpha_s + \alpha_{Age}(A(i) - \bar{A})$$  \hfill (S7)
The mean total number of children is then
\[ \log(\lambda(i)) = \alpha_0 + \alpha_{\text{FWHR}}(F(i) - \mu_F) + \alpha_{\text{RANK}}(R(i)) + \alpha_{\text{REG}}(G(i)) + \alpha_{\text{BP}}(B(i)) + \log(S(i) + (1 - S(i))\psi(i)). \]  

(S8)

The final term reflects the fact that if a soldier died during the Winter War, their reproduction is truncated, so that the mean number of offspring is (with a slight abuse of the definition of variables) \( \lambda(i)\phi(i) \).

**Rank**

Rank is measured on an ordinal scale: it is ordered (enlisted – junior officer – senior officer) but without any natural distance between the three categories. We therefore used a proportional odds logistic regression model\(^1\). We coded the ranks as \( R(i) \in \{1,2,3\} \), and then model \( \eta(i,j) \), the probability that individual \( i \) is in class \( j \) or less:

\[ \log\left( \frac{\eta(i,j)}{1 - \eta(i,j)} \right) = \gamma_0(j) - \gamma_{\text{FWHR}}F(i), \]  

(S9)

with the constraint that \( \gamma_0(1) < \gamma_0(2) < \gamma_0(3) = \infty \).

**Hierarchical Components**

We assume that the each level of the rank, regiment and birthplace effects are each drawn from a distribution with a common variance, i.e.

\[ \alpha_{\text{RANK}}(r) \sim N(0,\sigma^2_{\text{RANK}}) \]  

(S10)

\[ \alpha_{\text{REG}}(g) \sim N(0,\sigma^2_{\text{REG}}) \]  

(S11)

\[ \alpha_{\text{BP}}(b) \sim N(0,\sigma^2_{\text{BP}}) \]  

(S12)

\[ \beta_{\text{RANK}}(r) \sim N(0,\tau^2_{\text{RANK}}) \]  

(S13)

\[ \beta_{\text{REG}}(g) \sim N(0,\tau^2_{\text{REG}}) \]  

(S14)

\[ \beta_{\text{BP}}(b) \sim N(0,\tau^2_{\text{BP}}) \]  

(S15)

**Missing Data**

A small amount of the data was missing: 11% of regiment data, 5% of rank data and 1% of birthplace data. These were imputed as additional variables, by making them categorical variables with Dirichlet distributions with equal prior weights if 1 for each class. One individual did not have face shape data, but the model above accounted for this missing data.

The individuals without regiment data may have come from another regiment than the three used here. We thus imputed their regiment effects by adding 5 extra regiments in the data, with no individual in them, so that the soldiers with no regiment would have a probability to be in one of these additional regiments. This allows us to average over possible regiment effects.
Model Fitting

The model was fitted with a Bayesian approach, using the following vague prior distributions:

\[
\begin{align*}
\sigma_R^2, \sigma_F^2, \sigma_{RANK}^2, \sigma_{REG}^2, \sigma_{BP}^2, \tau_{RANK}^2, \tau_{REG}^2, \tau_{BP}^2 &\sim U(0,100) \\
\alpha_0, \alpha_S, \beta_0 &\sim N(0,10+3) \\
\alpha_{FWHR}, \alpha_{AGE}, \beta_{FWHR}, \beta_{AGE}, \gamma_{FWHR} &\sim N(0,100) \\
\gamma_0(1) &\sim U(-100,\gamma_0(2)) \\
\gamma_0(2) &\sim U(\gamma_0(1),100)
\end{align*}
\]

The estimation was carried out by MCMC. Ten chains were run and after a burn-in of 10,000 iterations, a further 200,000 iterations were run, and the output thinned to every fifth iteration, giving a total of 400,000 iterations. Convergence and mixing were judged by eye from plots of the chain histories: the posteriors for all parameters had an effective size over 1500 draws, and this was only under 10 000 for two standard deviations. The model was run in OpenBUGS through the Brugs package\(^2\).

The posterior distributions of factors (e.g. Rank) are summarised as their mean-centred effects, for example

\[
\beta_{RANK}^*(r) = \beta_{RANK}(r) - \frac{1}{3} \sum_{i=1}^{3} \beta_{RANK}(i).
\]

For Regiment, we only sum over the three known regiments.

Model Checking

Several aspects of the model were examined by fitting an expanded model: a single model with all of the following changes was fitted:

The first order interactions between FWHR, and Regiment and Rank were added (i.e. FWHR*(Regiment + Rank)):

\[
\begin{align*}
\alpha_{FWHR}(R(i),G(i)) &= \alpha_{FWHR}^0 + \alpha_{FWHR}^{RANK}(R(i)) + \alpha_{FWHR}^{REG}(G(i)) \\
\beta_{FWHR}(R(i),G(i)) &= \beta_{FWHR}^0 + \beta_{FWHR}^{RANK}(R(i)) + \beta_{FWHR}^{REG}(G(i))
\end{align*}
\]

The effects of angle of face \((A(i))\) and age when the photograph was taken \((P(i))\) on measured FWHR were added:

\[
\begin{align*}
R(i,m) &\sim N(\phi(i),\sigma_R^2) \\
\phi(i) &= F(i) + \phi_{ANGLE} A(i) + \phi_{SMILE} M(i) + \phi_{PHOTOAGE} P(i)
\end{align*}
\]

\(P(i)\) is a factor with three levels, so this effect was mean-centred. In addition, the main model was run for data with individuals with no regiment removed.

Model Checking Results

The estimated interactions between FWHR and Rank or Regiment are plotted in Fig. S2. There is no evidence for any differences: indeed the 95% HPDIs for all of the interactions considerably overlap zero. Similarly, there is no evidence for any effect of the angle of the
face to the camera and of age when the photograph was taken on the measures of FWHR.

Overall there was little effect on the survival model of removing the soldiers with no regiment (Fig. S3), except that the FWHR coefficient moved close to zero. There were larger differences in the model for the number of offspring, but they did not affect the relationship with FWHR.

References


**Figure S1**: Directed Acyclic Graph of model. Rectangles are data (i.e. known values), ovals are stochastic values. Arrows show dependencies, e.g. FWHR 1(i) depends on FWHR(i) and $\tau_{FACE}$. Indices show that values are a vector, e.g. FWHR 1(i) is indexed by i (for individual soldiers). Prior distributions and some hierarchical structure is omitted for clarity.
**Figure S2:** Estimated FWHR:Rank and FWHR:Regiment interactions on survival and number of offspring, and effects of angle of face to camera and age when photograph was taken on measures of FWHR. Vertical line: posterior mode, thick bar: 50% HPDI, thin line: 95% HPDI. Effects are mean-centred (i.e. the mean of the three levels for any interaction is zero).
**Figure S3:** Estimated parameters for model with all soldiers, and only those from the three regiments.