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### Author for correspondence:

Manoj Srinivasan

e-mail: [srinivasan.88@osu.edu](mailto:srinivasan.88@osu.edu)

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## Biomechanics

# Sideways walking: preferred is slow, slow is optimal, and optimal is expensive

Matthew L. Handford and Manoj Srinivasan

Mechanical and Aerospace Engineering, The Ohio State University, Columbus, OH 43210, USA

When humans wish to move sideways, they almost never walk sideways, except for a step or two; they usually turn and walk facing forward. Here, we show that the experimental metabolic cost of walking sideways, per unit distance, is over three times that of forward walking. We explain this high metabolic cost with a simple mathematical model; sideways walking is expensive because it involves repeated starting and stopping. When walking sideways, our subjects preferred a low natural speed, averaging  $0.575 \text{ m s}^{-1}$  (0.123 s.d.). Even with no prior practice, this preferred sideways walking speed is close to the metabolically optimal speed, averaging  $0.610 \text{ m s}^{-1}$  (0.064 s.d.). Subjects were within 2.4% of their optimal metabolic cost per distance. Thus, we argue that sideways walking is avoided because it is expensive and slow, and it is slow because the optimal speed is low, not because humans cannot move sideways fast.

## 1. Introduction

Humans walk and run in a manner that approximately minimizes metabolic energy expenditure [1–4]. However, most evidence for metabolic energy optimality has been for natural gaits, such as walking and running. Here, we examine optimality in an unnatural movement: a sideways walking gait (figure 1a). With human subject experiments, we show that the sideways walking speeds which humans prefer are close to minimizing the metabolic cost per distance. Both experiment and mathematical models suggest that the metabolic cost of sideways walking is significantly higher than normal walking. This high cost suggests why people rarely walk sideways even when they wish to move sideways.

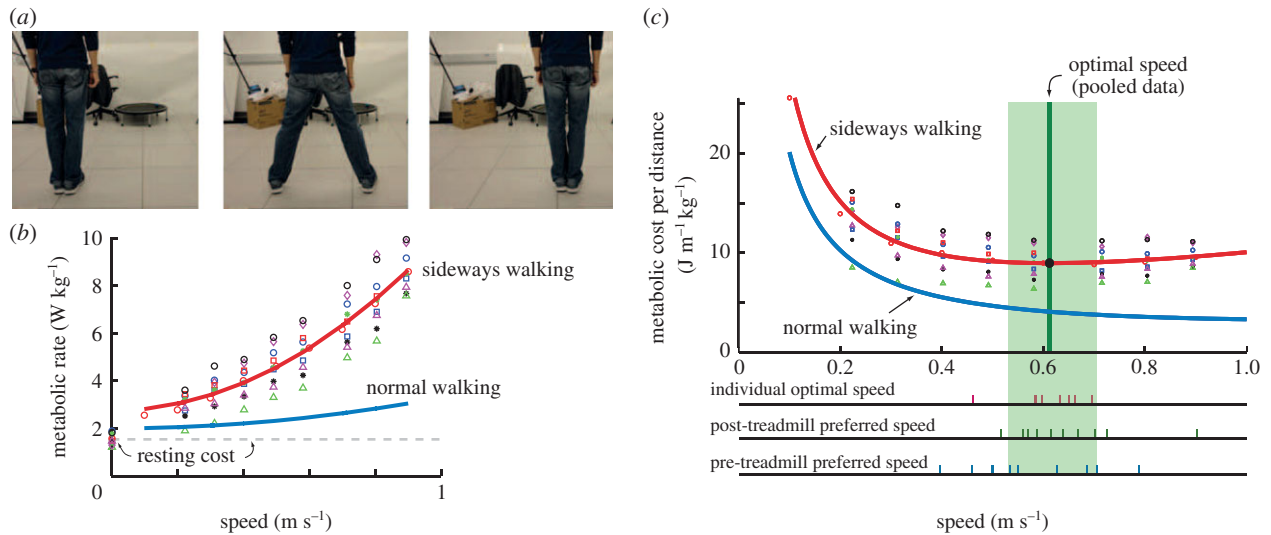
## 2. Material and methods

### (a) Experimental protocol

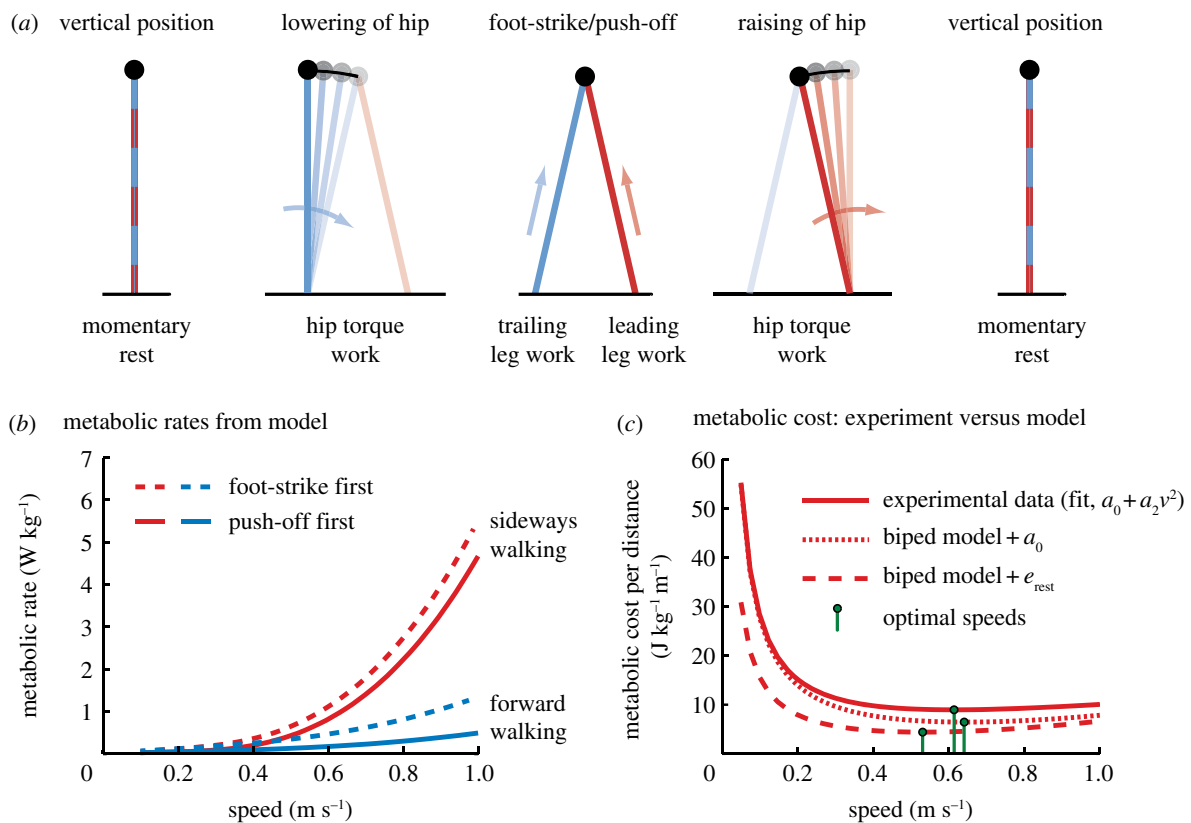
The protocol was approved by the Ohio State University's Institutional Review Board. Subjects gave informed consent. The subjects were 10 healthy adults (50% males: females), with mean age 27.5 years (12.5 s.d., range 19–52), mean height 173.8 cm (8.0 s.d.) and mean mass 73.2 kg (13.5 s.d.).

First, before extensive practice in walking sideways, the subjects were asked to walk sideways for 61 m in a hallway at a speed they found comfortable. They reversed direction mid-way (30.5 m), switching their leading leg. The 'pre-treadmill' preferred speed was estimated by timing and averaging four such trials.

Next, the subjects walked sideways on a treadmill, with their preferred leg leading. We used seven to nine treadmill trials (80 trials across 10 subjects), with speeds  $0.1\text{--}1.0 \text{ m s}^{-1}$ , the top speed adapted to the subject's comfort level. Metabolic rates were estimated with a metabolic measurement system (Oxycon Mobile, mass 1 kg), which measures respiratory oxygen and carbon dioxide flux. We approximate the metabolic rate per unit mass in  $\text{W kg}^{-1}$  by  $\dot{E} = 16.58 \dot{V}_{\text{O}_2} + 4.51 \dot{V}_{\text{CO}_2}$ , where  $\dot{V}$  is in  $\text{ml s}^{-1} \text{ kg}^{-1}$  [5]. Each treadmill trial lasted 6 min: 3 min to reach a steady state and 3 min to estimate an average steady state metabolic rate. Resting metabolic rate was measured while sitting before the treadmill trials.



**Figure 1.** Sideways walking experiment and energetics. (a) Sideways walking gait. Subjects faced perpendicular to their travel direction, never crossed their feet and always had at least one foot on the ground. (b) Mass-normalized metabolic rate for all subjects (different markers) and a quadratic speed-dependence fit to the pooled data from all subjects (red curve). Metabolic rate for normal (forward) walking [2] is also shown. (c) Mass-normalized metabolic cost per unit distance for sideways walking. The best fit curves for sideways walking (red line) and normal walking (blue line [2]) are shown. The dark green vertical line indicates the optimal speed ( $0.612 \text{ m s}^{-1}$ ) from the pooled data. The light green band ( $0.531\text{--}0.705 \text{ m s}^{-1}$ ) is the range of speeds with less than a 1% increase over the optimal metabolic cost per distance. Displayed below the graph are the subjects' optimal speeds, post-treadmill preferred speeds and pre-treadmill preferred speeds.



**Figure 2.** Sideways walking model. (a) An idealized biped with the upper body mass centred at the hip, performing idealized sideways walking. The sequence of hip lowering, foot-strike and push-off impulses, hip raising and vertical rest phases is shown. (b) The metabolic rates of sideways and forward walking obtained from the simple mathematical models with foot-strike preceding push-off and vice versa for fixed step length  $d_{\text{step}} = \ell/2$  and  $\ell = 1 \text{ m}$ ; no leg-swing or resting cost. (c) Sideways walking metabolic cost per distance from experiment and model (two versions) with leg-swing cost and unconstrained (optimally chosen) step lengths. The optimal speeds from experiment and model are shown.

After the treadmill trials, the preferred speeds were measured again in the hallway. During the hallway trials, the subjects wore the metabolic equipment, but no metabolic measurements were made. We do not discuss the post-treadmill preferred speeds because subjects had different cool-down protocols after the treadmill trials.

## (b) Mathematical model

We consider a simple mathematical model of a biped (figure 2a), similar to point-mass models used for forward walking [6,7]. This biped has the upper body mass  $m$  centred at the hip, with high moment of inertia, so that the hip muscles can torque the leg forward by reacting against this upper body. During single

stance, ignoring the swing leg, the biped is an inverted pendulum with a clockwise hip torque  $\tau$ , described by the equations:  $m\ell^2\dot{\theta} + mg\ell \sin \theta = \tau$ , with leg angle  $\theta$  measured clockwise from the vertical ( $\theta = 0$ ), acceleration due to gravity  $g$  and leg length  $\ell$ .

For this biped model, an idealized sideways walking gait is described in figure 2a (see also the electronic supplementary material, animation). This gait starts and returns every stride to a vertical position with angular rate  $\dot{\theta} = 0$ . Starting at the vertical position, finite positive work is performed by a hip-torque impulse, taking the angular rate  $\dot{\theta}$  from zero to some finite quantity instantaneously. The inverted pendulum then coasts passively, lowering the hip, until a foot-strike and push-off impulse achieve a step-to-step transition. The inverted pendulum coasts passively, raising the hip until the leg is vertical again, when a hip-torque impulse decelerates  $\dot{\theta}$  back to zero.

Consider walking with average forward speed  $v$ , step length  $d_{\text{step}}$  and step period  $T_{\text{step}} = d_{\text{step}}/v$ . For lowering the hip, the initial leg angle at  $t = 0$  is  $\theta(0) = 0$ . We solve for the initial angular rate  $\dot{\theta}(0^+)$  just after the accelerating torque impulse, so that the passive inverted pendulum motion reaches the step-to-step transition leg angle  $\theta(T_{\text{step}}/2) = \alpha = \sin^{-1}(d_{\text{step}}/2\ell)$  in half the step period,  $T_{\text{step}}/2$ . The hip speed at this time is  $v^- = \ell\dot{\theta}(T_{\text{step}}/2)$ .

The initial kinetic energy  $\frac{1}{2}m\ell^2\dot{\theta}(0^+)^2$  is the positive work performed by the hip-torque impulse. The foot-strike and push-off impulses perform negative and positive work of equal magnitude:  $\frac{1}{2}m|v^-|^2(1 - \cos^2 2\alpha/\cos^2 \alpha)$  when foot-strike precedes push-off and  $\frac{1}{2}m|v^-|^2 \tan^2 \alpha$  when push-off precedes foot-strike. Raising the hip is the reverse of lowering the hip, switching positive and negative work. Metabolic cost is modelled as a weighted sum of positive and negative work, scaled by their respective efficiency reciprocals,  $b_1 = 4$  and  $b_2 = 1$  [7]. In addition to this metabolic cost, for some comparisons, we added a leg-swing cost and a constant metabolic rate, equal to  $e_{\text{rest}}$  or  $a_0$ , to the model (otherwise, model has zero metabolic rate at zero speed); for these comparisons, we found the optimal step length and metabolic cost at every speed (see electronic supplementary material, S3).

We performed analogous calculations for forward walking, one model with foot-strike (usually called heel-strike [7–9]) before push-off and another with the impulse sequence reversed [7,9]. Forward walking models differ from sideways walking models only in that they do not have a vertical rest position at each step. See the electronic supplementary material for a detailed derivation, methods and computer programs.

### 3. Results

The resting metabolic rate per unit mass,  $e_{\text{rest}}$ , averaged  $1.542 \text{ W kg}^{-1}$  (s.d. 0.222). The sideways walking metabolic rate increased monotonically with speed  $v$  (figure 1b). Similar to normal walking [2,4,10], the total sideways walking metabolic rate per unit mass,  $\dot{E}$  (including the resting cost), is approximated well, using least squares, by:  $\dot{E} = a_0 + a_2v^2$ .

For metabolic rate data pooled over the subject population (figure 1b),  $a_0 = 2.742 \text{ W kg}^{-1}$  and  $a_2 = 7.313 \text{ W (m s}^{-1})^{-2} \text{ kg}^{-1}$ . Fitting the data from individual subjects separately and then averaging the coefficients gives:  $\bar{a}_0 = 2.746 \text{ W kg}^{-1}$  and  $\bar{a}_2 = 7.306 \text{ W (m s}^{-1})^{-2} \text{ kg}^{-1}$ . See the electronic supplementary material, S4 for these coefficients' error estimates. Note that zero-speed cost  $a_0 > e_{\text{rest}}$ , as also found for forward walking ([2,4]; electronic supplementary material, S5).

The total metabolic cost per unit distance per unit mass is given by  $E' = \dot{E}/v = a_0/v + a_2v$  (see [10]), shown in figure 1c. The speed that minimizes  $E'$  is given by

$$v_{\text{opt}} = \sqrt{a_0/a_2} = \sqrt{2.742/7.313} = 0.612 \text{ m s}^{-1} \text{ (s.d. 0.027; 95\% CI 0.495–0.754 m s}^{-1}\text{)}.$$

By subtracting resting cost, we get the net metabolic rate  $\dot{E}_{\text{net}} = \dot{E} - e_{\text{rest}}$ . The net metabolic cost per unit distance  $E'_{\text{net}} = \dot{E}_{\text{net}}/v$  is minimized at speed  $v_{\text{opt,net}} = \sqrt{(a_0 - e_{\text{rest}})/a_2} = 0.405 \text{ m s}^{-1}$ . Not subtracting the resting cost gives the 'maximum range speed', the speed which maximizes distance for given energy [10]. For forward walking,  $v_{\text{opt}}$  (about  $1.3\text{--}1.4 \text{ m s}^{-1}$ ) is much closer to the preferred walking speeds than  $v_{\text{opt,net}}$  (about  $0.7\text{--}0.9 \text{ m s}^{-1}$ ); see [10] for a review of these issues. Here, we use  $v_{\text{opt}}$  as the predicted optimal speed.

The subject-specific optimal speeds, obtained from individual fits, averaged  $0.610 \text{ m s}^{-1}$  (s.d. 0.064). Preferred hallway walking speeds before and after the treadmill trials averaged  $0.575 \text{ m s}^{-1}$  (s.d. 0.123) and  $0.649 \text{ m s}^{-1}$  (s.d. 0.111), respectively. These speeds are shown in figure 1c.

The metabolically optimal speeds and the pre-treadmill preferred speeds have similar means, differing only by  $0.035 \text{ m s}^{-1}$ . The subjects' individual optimal speeds differed from their pre-treadmill speeds by a mean absolute difference of  $0.12 \text{ m s}^{-1}$ , corresponding to an increase of 2.4% in  $E'$  over optimal. Furthermore, out of the 10 subjects, five chose a speed with an  $E'$  within 1% of the optimal (the green band in figure 1c). Thus, the subjects are not far from optimal.

From prior research, forward walking has  $a_0 \approx 2.1 \text{ W kg}^{-1}$  and  $a_2 \approx 1\text{--}1.5 \text{ W (m s}^{-1})^{-2} \text{ kg}^{-1}$  [2,4]. For matched speeds below  $1 \text{ m s}^{-1}$ , the net metabolic rate  $\dot{E}_{\text{net}}$  of sideways walking is three to five times that of forward walking. The optimal sideways walking speed ( $0.612 \text{ m s}^{-1}$ ) is only half as fast as the optimal forward walking speed ( $1.25\text{--}1.35 \text{ m s}^{-1}$  [2,10]). At their respective optimal speeds, the optimal sideways metabolic cost per distance  $E'$  ( $8.95 \text{ J m}^{-1} \text{ kg}^{-1}$ ) is about three times the optimal forward  $E'$  ( $3.2 \text{ J m}^{-1} \text{ kg}^{-1}$ ).

Our mathematical model of sideways walking also has a much higher cost than similar inverted pendulum models of forward walking, thus qualitatively explaining the vast difference in measured costs (figure 2b, ignoring leg-swing cost). In these models, compared with optimal inverted pendulum forward walking [7,9], the metabolic rate of sideways walking is about 3–10 times higher at  $1 \text{ m s}^{-1}$  depending on whether foot-strike or push-off is assumed to occur first for both gaits; the costs are similar at infinitesimal speeds. While the model with leg-swing cost still underestimates the metabolic cost (figure 2c), it predicts an optimal speed of  $0.5319 \text{ m s}^{-1}$  when  $e_{\text{rest}}$  is added and  $0.6420 \text{ m s}^{-1}$  when  $a_0$  is added (see the electronic supplementary material, S3).

### 4. Discussion

Humans likely prefer a slow sideways walk because of the low optimal speed. While subjects stayed within 1–2% of optimal metabolic cost at their preferred speeds, their preferred speeds were highly variable. This variability is perhaps inevitable given the insensitivity of  $E'$  to speed, near the optimal speed (low curvature). Such insensitivity, if typical, might make predicting human coordination from metabolic optimality inaccurate [7], even if humans were close to optimal. However, this speed variability may decrease with sufficient practice.

Slow speeds in humans with mobility issues is sometimes implicitly attributed to an inability to walk faster (e.g. [11]).

However, as argued here for sideways walking, humans walk close to their (slow) metabolically optimal speeds even while using prostheses and crutches [2].

Sideways walking has a high metabolic cost e.g. at  $1 \text{ m s}^{-1}$ , it has the same gross metabolic rate as running at  $2.3 \text{ m s}^{-1}$  (see the electronic supplementary material, S1, for other comparisons). Nevertheless, humans often use one or two sideways steps when they wish to move sideways a short distance e.g. while working at a kitchen counter or self-organizing for a group photograph. But when humans wish to move sideways for longer distances, they simply turn and walk facing forward. Here, the one-time turning cost is compensated by the substantial cost reduction by walking forward. Measuring the turning cost will let us predict the distance under which humans step sideways as opposed to turn and walk forward.

In our mathematical model, we used a work-based metabolic cost, found appropriate also for forward human walking [8,12], neglecting force-related terms [12,13], which may explain the model's underestimation. The model's work estimates are an approximate lower bound for the

leg work and will require a corresponding metabolic cost, assuming negligible elastic recovery mechanisms.

Sideways walking involves no knee flexion and the ankle pronation–supination is much smaller than the plantarflexion–dorsiflexion in forward walking. Given such kinematic simplicity, sideways walking may serve as a simpler task for studying locomotion energetics, perhaps requiring simpler muscle-driven human models than necessary for forward walking. Inverted pendulum-like walking models may also be better suited to sideways walking than forward walking. Studying such simplified tasks might bridge the gap between studying isolated limb movements such as leg swing and studying normal walking.

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